Reg. No. : [| | | | | | | | | | |]

Question Paper Code : 80209

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Second Semester

Civil Engineering

MA 8251 — ENGINEERING MATHEMATICS – II

(Common to All branches (Except Marine Engineering))

(Regulation 2017)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If λ is the eigenvalue of the matrix A, then prove that λ^2 is the eigenvalue of A^2.

2. If the eigenvalues of the matrix A of order 3×3 are 2, 3 and 1, then find the determinant of A.

3. Find the unit normal vector to the surface $x^2 + y^2 = z$ at $(1, -2, 5)$.

4. State Stoke's theorem.

5. Is the function $f(z) = e^z$ analytic.

6. Find the fixed point of the bilinear transformation $w = \frac{1}{z}$.

7. Evaluate $\int_C \sin z \, dz$, where C is the entire complex plane.

8. Define singularity of a function $f(z)$.

9. Find $L[e^{-t} \sin t]$.

10. State sufficient conditions for the existence of Laplace transform.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Find the eigenvalues and the eigenvectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem find A^{-1}, if $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $2xy - 2yz + 2xz$ into a canonical form by an orthogonal reduction. (16)

12. (a) (i) Verify Gauss divergence theorem for the vector function $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cuboids bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 1$, and $z = 1$. (10)

(ii) Find the value of n so that the vector $r^n\vec{r}$ is irrotational and solenoidal. (6)

Or

(b) (i) Apply Green's theorem to evaluate $\int_C \left[\left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy\right]$, where C is the boundary of the area by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$. (8)

(ii) Verify Stoke's theorem for $\vec{F} = \left(x^2 + y^2\right)\vec{i} + 2xy\vec{j}$, taken around the rectangle bounded by the lines $x = 0, y = 0, x = 1$ and $y = 1$. (8)

13. (a) (i) Determine the analytic function $f(z) = u + iv$, if $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$. (8)

(ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto $w = i,\ 0, -i$. (8)

Or

(b) (i) Show that the real and imaginary parts of an analytic functions are harmonic. (8)

(ii) Find the image of $|z - 2i| = 2$ under the transformation $w = \dfrac{1}{z}$. (8)

14. (a) (i) If $F(a)=\oint_C \frac{(3z^2+7z+1)}{z-a}dz$, where C is $|z|=2$, then find $F(1-i)$ and $F'(1-i)$. (8)

(ii) Using contour integration, evaluate $\int_0^{\infty}\frac{dx}{(x^2+1)^2}$. (8)

Or

(b) (i) Obtain the Laurent's series expansion of $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ if $2<|z|<3$. (8)

(ii) Evaluate by using contour integration $\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$. (8)

15. (a) (i) Find the Laplace transform of $f(t)$ with period $2a$, where $f(t)=\begin{cases} t, & \text{for } 0<t<a \\ 2a-t, & \text{for } a<t<2a \end{cases}$. (8)

(ii) Using convolution theorem, find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$. (8)

Or

(b) (i) Find $L\left[\frac{\cos 2t-\cos 3t}{t}\right]$. (8)

(ii) Solve $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=e^{3t}$; given that $y(0)=0, \frac{dy}{dt}(0)=0$. (8)
